FLIGHT PATH OPTIMIZATION FOR COMPETITION SAILPLANES THROUGH STATE VARIABLES PARAMETERIZATION

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Abstract. *This paper presents a numerical process for determination of optimal flight paths for competition soaring. The issue is reduction of flight time in order to soar towards an ascending thermal and climb, through thermal flying, to the initial altitude. The optimization procedure consists in the application of a Direct Method in order to obtain suboptimal solutions through parameterization of state variables, unlike a previous study by the same authors which was based on control parameterization. A mathematical programming procedure is used in order to determine the sub-optimal values for the parameterized state variables. The optimal control law, which is necessary for the generation of the sub-optimal state, is obtained through a step by step penalty technique. Besides this numerical approac, an alternative anayitical approach for this optimization problemis is presented, taking in acount the dynamic transition between straight and circular flight, more accurate than the classical MacCready solution. The numerical and analitical results are compared proving its coerence. The obtained results demonstrate that the optimization of transitory phases is important for the minimization of total flight time.*

Keywords*: Sailplane, Flight Path Optimization*

1. Introduction

The classic problem of competition sailplane flight trajectory (Figure 1) consists in minimizing the time spent by the sailplane to flybetween two thermals (A-C) and climb to the initial altitude (C-E) (Weinholtz, 1967; Reichamnn, 1978, 1980). The classic solution for this problem, presented in 1952 by the North-American Engineer Paul MacCready, is based on an equilibrium analysis which doesn't take into account the transitory effects during the trajectory.

Other authors (Vanderbei, 2000; Goto and Kawabe, 1994,1999; Dickmanns, 1981; De Jong, 1981; Pierson and De Jong, 1978) presented studies using dynamic models which take into account the transitory effects of the problem; however, their models were simplified. Recently, preliminary results were presented regarding optimization of a sailplane flight path, based on a dynamic model for symmetric flight, without analysis of climbing flight (Iscold and Pinto, 2003).

This paper, a continuation of the study presented by Iscold and Pinto (2003), shows the approach of the entire problem, taking into consideration: i) the acceleration phase as the sailplane leaves the thermal (pitch down); ii) the deceleration phase when the sailplane enters the thermal (pitch up) and iii) the phase of climbing within the thermal. Optimization is reached through parameterization of state, unlike the previous paper of these authors which was based on control parameterization.

2. The problem, including transitory and climbing phases

The complete problem to be analyzed in this paper can be presented as seen in Figure 1. According to this figure, the optimization process can be written as:

$$\min\left[t_{AB}+t_{BC}+t_{CD}+t_{DE}\right] \tag{1}$$

subject to:

$$\begin{aligned} & V \le VNE; \\ & n_z \le \left(n_z\right)_{\max} \\ & \max_{0<x<Xf}\left[y(0)-y(x)\right] \le h \end{aligned} \tag{2}$$

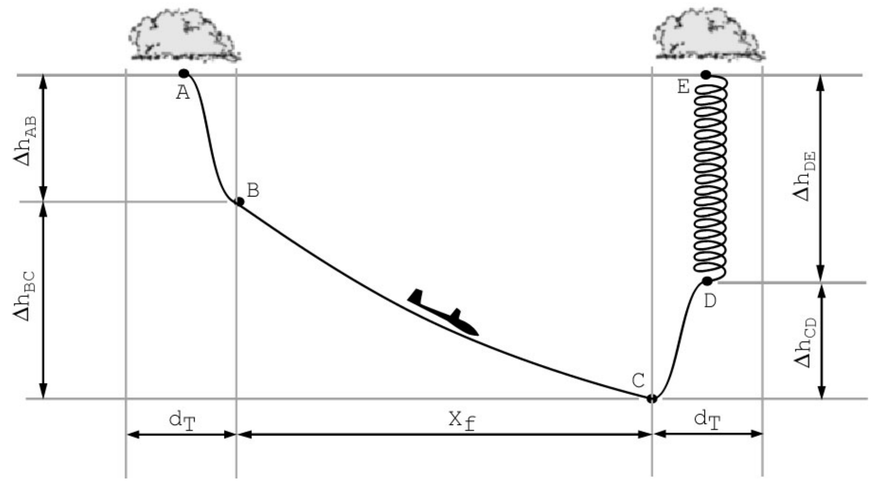

Figure 1 – Sailplane trajectory optimization problem with transitory and climbing phases

The three first terms on equation (1) represent, respectively, time spent during the steps acceleration, soaring, and deceleration. The fourth term represents the time spent in climbing phase. The inequality constraints represent the upper operational limit of sailplane velocity (VNE), the vertical weight limit (n_z) and the imposition that the initial altitude of the sailplane (h) be greater than the largest altitude loss.

3. Dynamic model

The dynamic model used in this paper is the same one presented by Iscold and Pinto (2003), modified by the addition of simplified equations to represent the dynamic of the turning flight of the sailplane (Thomas, 1999). As in Iscold and Pinto, 2003, the state variables are:

$$\begin{aligned} x_1 &= x \\ x_2 &= y \\ x_3 &= \theta \\ x_4 &= \dot{x} = V_X \\ x_5 &= \dot{y} = V_Y \\ x_6 &= \dot{\theta} = q \end{aligned} \qquad (3)$$

The sailplane motion equations are:

$$\begin{aligned} \dot{x}_1 &= x_4 \\ \dot{x}_2 &= x_5 \\ \dot{x}_3 &= x_6 \\ \dot{x}_4 &= \frac{1}{m}\left[L\sin\eta - D\cos\eta + L_T \sin\gamma\right] \\ \dot{x}_5 &= \frac{1}{m}\left[-L\cos\eta\cos\phi - D\sin\eta\cos\phi + L_T\cos\gamma\cos\phi + W\right] \\ \dot{x}_6 &= \frac{1}{J}\left[M + L\,x_A\cos\alpha - L_T\,x_T\cos\alpha_T\right] \end{aligned} \qquad (4)$$

Notice that the previous equations can be used to analyze simplified circular movement with small angular acceleration, which allows the analysis of thermal climb flight.

4. Optimal control problem solution

In order to solve the optimization problem, it is assumed that the flight path is composed by the following phases (Figure 2):

i) Starting from the climb flight velocity in the thermal (V_C), the sailplane must accelerate until to reach the velocity (V). This flight phase involves a pitch down acceleration of the aircraft;

ii) Once the velocity (V), is reached, the sailplane must fly in straight flight (soaring), in other words, with constant velocity;

iii) Once soaring is through, the sailplane must decelerate (pitch up) until it reaches, once again, the climb flight velocity within the thermal (V_C).

iv) Until the entire lost altitude during the trajectory is regained, the sailplane must maintain climb flight within the thermal with a constant velocity (V_C).

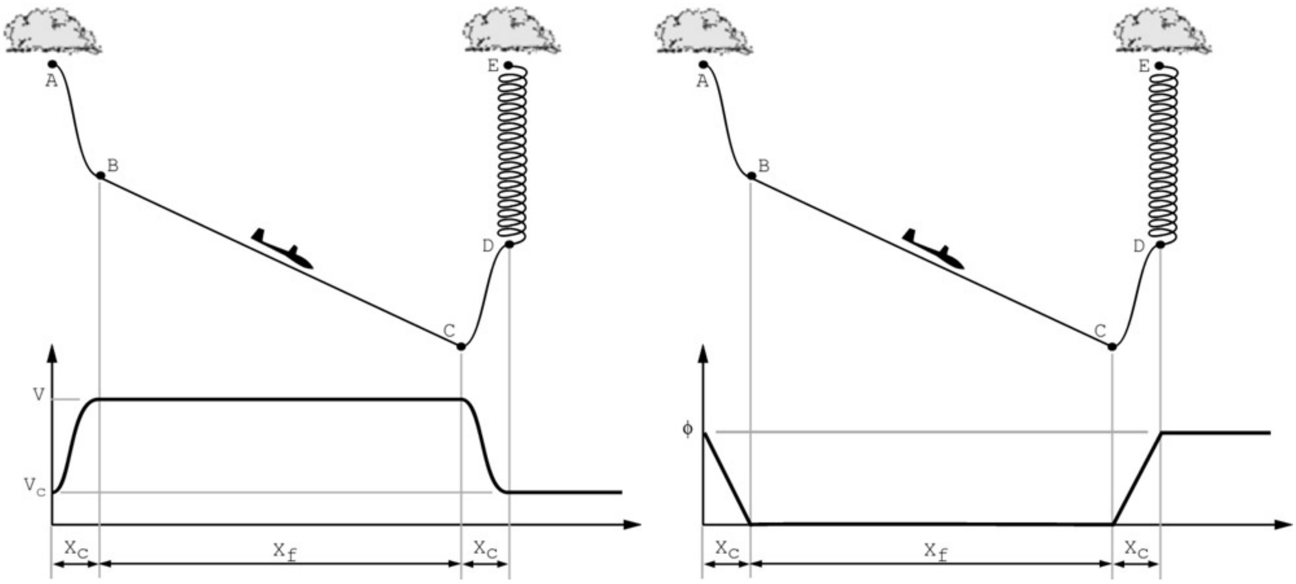

Figure 2 – Typical velocity and bank angle profile

For parameterization, it is assumed that the velocity evolution during the acceleration ($\overline{AB}$) and deceleration ($\overline{CD}$) will occur according to cubic polynomials. The cubic interpolations are performed by cubic Hermite polynomials. Then, the coefficients to be determined represent the velocity and the respective derivative values in the beginning and in the end of each phase (Figure 3). Pinto (1982) presents an approximation of the third degree using Hermite polynomials as:

$$y(\xi) = N_1(\xi)\alpha_0 + N_2(\xi)\alpha_1 + N_3(\xi)\beta_0 + N_4(\xi)\beta_1 \tag{5}$$

where:

$$N_1(\xi) = 2\xi^3 - 3\xi^2 + 1 \tag{6}$$

$$N_2(\xi) = -2\xi^3 + 3\xi^2 \tag{7}$$

$$N_3(\xi) = \xi^3 - 2\xi^2 + \xi \tag{8}$$

$$N_4(\xi) = \xi^3 - \xi^2 \tag{9}$$

where:

$$\xi = \frac{x - x_0}{x_1 - x_0} \tag{10}$$

$$\alpha_0 = y_0 \tag{11}$$

$$\alpha_1 = y_1 \tag{12}$$

$$\beta_0 = \left.\frac{dy}{dx}\right|_0 \tag{13}$$

$$\beta_1 = \left.\frac{dy}{dx}\right|_1 \tag{14}$$

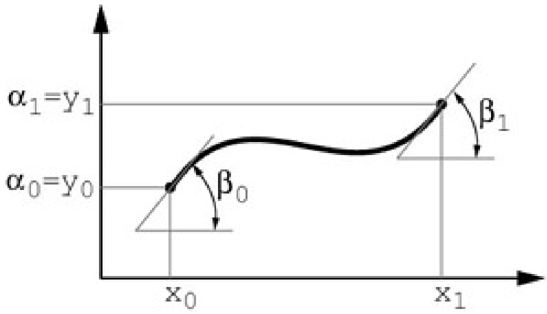

Figure 3 – Hermite polynomial parameters

As mentioned earlier, the dynamic model adopted contemplates, in a simplified manner, the sailplane flying in curve, which depends of the bank angle (ϕ). Therefore, it is necessary to determine a variation law for bank angle during the acceleration and deceleration phases. In the present paper, a liner evolution of the bank angle is adopted, as shown in Figure 2. This profile introduces rolling velocities which are compatible with maneuver capabilities of typical sailplanes.

5. Elevator deflection law

The elevator deflection law must be obtained along the numerical integration, step by step, as the one that minimizes the difference between the sailplane's flight velocity and a pre-established velocity. Also, it is important to "teach" the numerical integrator the direction of the velocity variables. This is possible by adding to the objective function a term that corresponds to the condition of tangency to the flight trajectory. Therefore, for each integration step, it is necessary to find the elevator angle (η) which minimizes the function:

$$J(\eta) = k_1\left[V(\eta) - \bar{V}\right]^2 + k_2\left[V'(\eta) - \bar{V}'\right]^2 \tag{15}$$

where V and V' denote, respectively, the sailplane flight velocity and its derivative with respect to the state variable x_1, while $\bar{V}$ and $\bar{V}'$ denote the respective pre-determined values.

The constants k_1 and k_2 represent weights which must be chosen appropriately. For this paper, was successfully adopted:

$$k_1 = k_2 = 1 \tag{16}$$

Notice that optimal elevator angle (η) can be found through a unidirectional search method. For this paper a procedure based on the Golden Section Method was chosen (Luenberger, 1984).

6.Optimization of flight trajectory

When the velocity profile shown in Figure 2 is adopted, one will have, initially, the following parameters to be optimized: i) the flight velocity during the climbing (V_C); ii) soaring velocity (V); iii) acceleration distance (X_0); iv) decelaration distance (X_1); v) the velocity derivatives in the cubic extremes ($\bar{V}_0', \bar{V}_1', \bar{V}_2', \bar{V}_3'$). However, in order to smoothen velocity profiles, it was imposed that $\bar{V}_0' = \bar{V}_1' = \bar{V}_2' = \bar{V}_3' = 0$.

In addition, the optimal flight velocity during climb flight (V_C) can be determined separately through a statistical analysis of the thermal rising flight problem (Thomas, 1999). Therefore, during the optimization procedure, this velocity is determined a priori.

Therefore, three remaining optimization variables (V, X_0 and X_1) were determined through a mathematical programming algorithm. This problem has been shown to be stable and easier than it seems, once, as shown through experiments, optimal V, X_0 and X_1 can be determined almost independently.

7. Analitical solution

In order to establish a simplified analytical solution, the velocity profile of the flight, between thermals was considered to be with linear variation between points A and B (Figure 2) and points C and D. Therefore the total time spent in flight between thermals will be the sum of the time spent in acceleration (t_1), the time spent in straight flight (t_2), the time spent in deceleration (t_3) and the time in circular thermal flight (t_4). Then, considering this velocity profile, it can be written that:

$$t_1 = \frac{X_0}{V_M} = \frac{2X_0}{\left(V_p + V_c\right)} \tag{17}$$

$$t_2 = \frac{X_f}{V_p} \tag{18}$$

$$t_3 = \frac{X_1}{V_M} = \frac{2X_1}{\left(V_p + V_c\right)} \tag{19}$$

$$t_4 = \frac{\Delta h}{V_T} \tag{20}$$

The total height spent during the flight between thermals can be evaluated by considering: i) the height spent in acceleration, considering the loss due potential to kinetic energy change (Δh_1) and the gain due thermal velocity (Δh_4); ii) the height spent in glide flight (Δh_3) and iii) the height gain in deceleration flight, considering the potential gain due to kinetic energy change (Δh_2) and thermal velocity (Δh_5). Thus, it can be easily determined that:

$$\Delta h_{1,2} = \frac{\left(V_0^2 - V_1^2\right)}{2g} - \left[\frac{\rho S C_{D0} X}{4mg\left(V_1^2 - V_0^2\right)}\left(V_1^4 - V_0^4\right) + k_1 X + \frac{4k_2 W X}{\rho S\left(V_1^2 - V_0^2\right)}\ln\left(\frac{V_1}{V_0}\right)\right] \tag{21}$$

$$\Delta h_3 = t_2 V_y\left(V_p\right) = \frac{X_f}{V_p} V_y\left(V_p\right) \tag{22}$$

$$\Delta h_4 = -\frac{t_1 V_T}{2} = -\frac{X_0 V_T}{V_p + V_c} \tag{23}$$

$$\Delta h_5 = -\frac{t_3 V_T}{2} = -\frac{X_1 V_T}{V_p + V_c} \tag{24}$$

Therefore it is possible to write an approximated equation for the time spent during a flight between thermals, including the time spent to climb to initial altitude. This equation can be derived in relation to the flight speed in order to determine the optimum condition for this flight, as:

$$\frac{\partial V_y\left(V_p\right)}{\partial V_p} = \frac{V_T + V_y\left(V_p\right)}{V_p} - \frac{V_p V_T}{X_f}\left[\frac{3X_t}{\left(V_p + V_c\right)^2}\right] - \left[\begin{array}{l} \dfrac{\rho S C_{D0} X_t}{2mgX_f}\dfrac{4V_p^4\left(V_p^2 - V_c^2\right) - 2V_p^2\left(V_p^4 - V_c^4\right)}{\left(V_p^2 - V_c^2\right)^2} \\ + \dfrac{8k_2 W X_t}{\rho S X_f}\dfrac{\left(V_p^2 - V_c^2\right) - 2V_p^2\ln\left(\dfrac{V_p}{V_c}\right)}{\left(V_p^2 - V_c^2\right)^2} \end{array}\right] \tag{25}$$

The results obtained with this equation will be showed and analyzed in the next section.

8. Results Analisys

This procedure was applied for the optimization of the trajectory of a PIK-20-B sailplane with a wing load of 31.2kgf/m^2 (Johnson 1978, Pinto et all, 1999), with the distance between thermals (X_f) ranging from 2000m to 16000m and thermal intensity (IT) of 2m/s and 5m/s. The thermal profile adopted was:

$$V_T = \frac{IT}{2}\left[1 + \cos\left(\frac{\pi r}{R}\right)\right] \tag{26}$$

where R denotes the radius of the thermal ($R = 250m$ was adopted).

Table 1 presents the optimal results obtained for the thermal intensities of 2m/s and 5m/s., and Table 2 presents the comparison between optimal and usual flight time between thermals. Figure 4 shows a typical trajectory (distance between thermals of 2000m and thermal intensity of 5m/s) obtained through the optimization procedure, where one can observe the optimal trajectory and the respective curves of: flight velocity, elevator deflection, load factor and mechanical energy (potential and kinetic) of the aircraft.

Table 1 – Optimal results for *IT* = 2m/s and *IT* = 5m/s

	IT = 2m/s				*IT* = 5m/s			
	X_0[m]	**X_1[m]**	**V[m/s]**	**t[sec]**	**X_0[m]**	**X_1[m]**	**V[m/s]**	**t[sec]**
2000	100	300	30.12	171	125	300	38.86	93
4000	100	300	30.54	317	125	300	39.84	171
8000	100	300	30.62	610	125	300	40.08	327
16000	100	300	30.55	1194	125	300	40.18	638

Table 2 – Comparison between optimal and usual times *IT* = 2m/s and *IT* = 5m/s

	IT = 2m/s				*IT* = 5m/s			
	t_{opt}[sec]	**t_{MC}[sec]**	**Δ[sec]**	**Δ[%]**	**t_{opt}[sec]]**	**t_{MC}[sec]**	**Δ[sec]**	**Δ[%]**
2000	164	171	-7	-4.2	90	93	-4	-4.0
4000	303	317	-15	-4.9	165	171	-7	-4.1
8000	596	610	-13	-2.2	320	327	-6	-2.0
16000	1184	1194	-10	-0.8	632	638	-6	-0.9

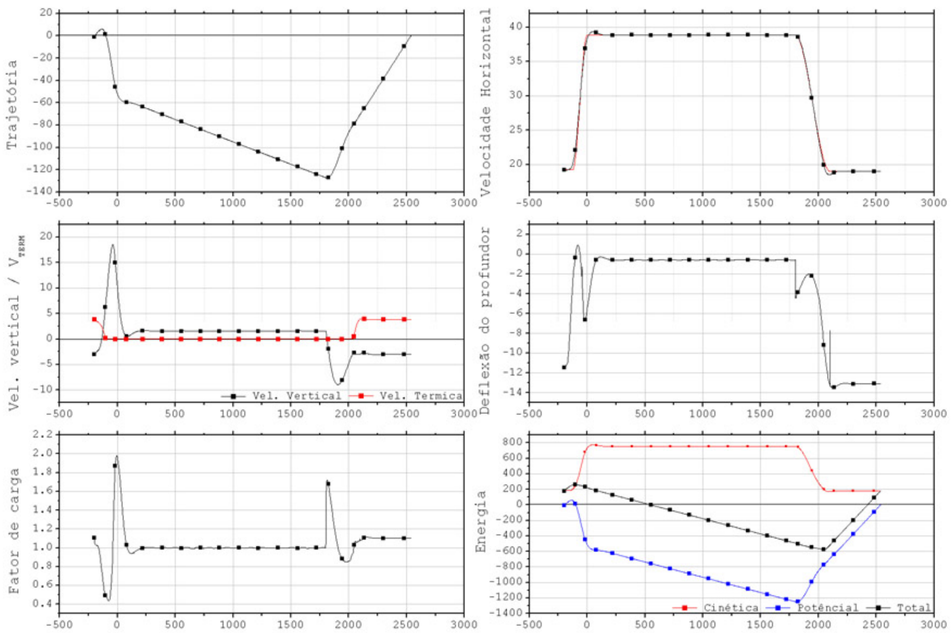

Figure 4 – Typical optimal trajectory – X_T=2000m e IT=5m/s

Table 3 shows a comparison between the optimal flight speeds obtained between the numerical and analytical procedures developed in this work and the optimal speed obtained by Mac Cready solution.

Table 3 – Comparison between optimal flight speeds [m/s]

	2000m					4000m					8000m					16000m				
IT	**1.5**	**2**	**3**	**4**	**5**	**1.5**	**2**	**3**	**4**	**5**	**1.5**	**2**	**3**	**4**	**5**	**1.5**	**2**	**3**	**4**	**5**
V_T	**0.34**	**0.72**	**1.50**	**2.27**	**3.04**	**0.34**	**0.72**	**1.50**	**2.27**	**3.04**	**0.34**	**0.72**	**1.50**	**2.27**	**3.04**	**0.34**	**0.72**	**1.50**	**2.27**	**3.04**
V_{num}	28.1	30.1	33.8	36.7	38.9	28.2	30.5	34.0	37.2	39.8	28.4	30.6	34.2	37.3	40.1	28.3	30.6	34.3	37.5	40.2
$V_{teorico}$	28.4	30.3	33.2	35.8	38.3	28.4	30.1	33.7	37.6	39.3	28.4	30.1	33.9	37.1	39.8	28.4	30.5	34.1	37.3	39.8
V_{MC2}	28.4	30.5	34.2	37.5	40.4	28.4	30.5	34.3	37.5	40.4	28.4	30.5	34.2	37.5	40.4	38.4	30.5	34.2	37.5	40.4

9. Results analisys

Notice (Table 1) that the optimal distances of acceleration and deceleration are not very sensitive to the variation in distance between thermals. The optimal deceleration distance, in particular, does not vary in relation to thermal intensity. This translates into the fact that, in practical terms, the acceleration and deceleration can be optimized separately. It is clear that the relative gain obtained with the proposed optimization procedure is greater for smaller distances between thermals. Indeed, the greater the distance between thermals, the smaller the relative participation of the transitory phases (acceleration and deceleration). It is interesting to observe, in Figure 4 that, for an optimal acceleration, the sailplane must gain some altitude in the beginning of the flight, reducing total altitude gain during the acceleration phase.

Also in Figure 4, it is shown that the load factor values associated to acceleration and deceleration maneuvers are within the sailplane operation limits, but are atypical if compared with usual values observed in such maneuvers.

Table 3 shows that the optimal results obtained by the numerical and analytical procedures proposed in this paper are very similar and these values in comparison to Mac Cready values tend to be smaller as the thermal strength increase. This observation is in accordance with the proposal of Schuemann (1972) that indicates that slightly smaller gliding speeds than MacCready speeds tend to give better results.

10. Conclusion

The optimization of competition sailplane flight trajectory was presented, including the acceleration and deceleration phases. Two procedures were studied, one numerical and one analytical.

The obtained numerical results, based on state parameterization, were compared to those of the Mac Cready theory and the usual acceleration and deceleration maneuvers. The advantages of the numerical procedure were significant, indicating that the practical considerations it takes into account are important.

The obtained analytical results, in comparison to numerical results, are shown to be coherent. The optimal speed obtained with numerical and analytical procedures in comparison to Mac Cready speed tend to be slightly smaller.

Comparative results indicate that the optimal time is sensitive to dynamic phases of flight (acceleration and deceleration phases) showing that special attention must be given to these phases.

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